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GLOBAL EXISTENCE FOR A STRONGLY COUPLED REACTION-DIFFUSION SYSTEMS WITH NONLINEARITIES OF EXPONENTIAL GROWTH

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ABSTRACT. The aim of this study is to construct the invariant regions in which we can establish the global existence of classical solutions for reaction-diffusion systems with a general full matrix of diffusion coefficients. Our techniques are based on invariant regions and Lyapunov functional methods. The nonlinear reaction term has been supposed to be of exponential growth.

1. Introduction

In this work, we are interested in global existence of classical solutions to the following reaction-diffusion system

$$\frac{\partial u}{\partial t} - a_{11}\Delta u - a_{12}\Delta v = f(u,v) \qquad \text{in} \quad (0,+\infty)\times\Omega, \tag{1.1}$$

$$\frac{\partial v}{\partial t} - a_{21}\Delta u - a_{22}\Delta v = g(u,v) \qquad \text{in} \quad (0,+\infty)\times\Omega, \tag{1.2}$$

with the initial conditions:

$$u(0,x) = u_0(x), \quad v(0,x) = v_0(x) \quad \text{in} \quad \Omega, \tag{1.3}$$

and the homogeneous boundary conditions:

$$\alpha u + (1-\alpha)\frac{\partial u}{\partial \nu} = 0, \;\; \alpha v + (1-\alpha)\frac{\partial v}{\partial \nu} = 0 \;\text{ on }\; (0,+\infty)\times\partial\Omega, \tag{1.4}$$

where Ω is an open bounded domain of class C^1 in $\mathbb{R}^n$, $\frac{\partial}{\partial\nu}$ denotes the outward normal derivative on $\partial\Omega$, α is a function of class C^1 on $\partial\Omega$ such that $0 \le \alpha \le 1$ and the diffusion terms $a_{ij},\ i,j = 1,2$ are supposed to be positive constants such that

$$(a_{12} + a_{21})^2 < 4a_{11}a_{22},$$

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which reflects the parabolicity of the system and implies at the same time that the matrix of diffusion

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

is positive definite. The eigenvalues λ_1 and λ_2 ($\lambda_1 < \lambda_2$) of A are positive.
If we put

$$\underline{a} = \min\{a_{11}, a_{22}\} \quad \text{and} \quad \overline{a} = \max\{a_{11}, a_{22}\}$$

then, the positivity of the diffusion terms implies that

$$\lambda_1 < \underline{a} \leq \overline{a} < \lambda_2.$$

We also put

$$\begin{aligned} \Sigma_1 &= \left\{(r,s) \in \mathbb{R}^2 : \quad \mu_2 r \leq s \leq \mu_1 r\right\}, \\ \Sigma_2 &= \left\{(r,s) \in \mathbb{R}^2 : \quad \frac{1}{\mu_2} s \leq r \leq \frac{1}{\mu_1} s\right\}, \end{aligned}$$

where

$$\mu_1 = \frac{a_{21}}{a_{11} - \lambda_1} > 0 > \mu_2 = \frac{a_{21}}{a_{11} - \lambda_2}. \tag{1.5}$$

We suppose:

(**A1**) f and g are continuously differentiable on $\Sigma_1 \cup \Sigma_2$,

(**A2**) $(-1)^j (f(r,s), g(r,s)) \in \Sigma_j$ and $\mu_i f(r, \mu_i r) = g(r, \mu_i r)$ for all $(r,s) \in \Sigma_i,\ i,j = 1,2\ (j \neq i)$,

(**A3**) $g(r,s) - \mu_j f(r,s) \leq (-1)^j \psi(s - \mu_j r)(g(r,s) - \mu_i f(r,s))$ for all $(r,s) \in \Sigma_i,\ i,j = 1,2\ (j \neq i)$,

where ψ is a nonnegative continuously differentiable function on $[0, +\infty)$ such that there exists a constant $\beta \geq 1$ satisfying $\lim\limits_{\eta \to +\infty} \eta^{\beta - 1} \psi(\eta) = \ell$ where ℓ is a nonnegative constant,

(**A4**) $g(r,s) - \mu_j f(r,s) \leq C\varphi((-1)^i (s - \mu_i r)) e^{\alpha (s - \mu_j r)^\beta}$ for all $(r,s) \in \Sigma_i,\ i,j = 1,2\ (j \neq i)$,

where $C > 0$, $\alpha > 0$, β is the same as in (**A3**) and φ is any nonnegative continuously differentiable function on $[0, +\infty)$ such that $\varphi(0) = 0$.

The initial data are assumed to be in Σ where $\Sigma = \Sigma_1$ or $\Sigma = \Sigma_2$.

The present investigation is a continuation of results obtained in [24]. In this study, we will treat the case of a general full matrix of diffusion coefficients and prove that if f and g satisfying (**A1**)-(**A4**), then Σ is an invariant region for problem (1.1)-(1.4). Once the invariant regions are constructed, we demonstrate that for any initial data in Σ satisfying

$$\|\mu_i u_0 - v_0\|_\infty < \frac{8\lambda_1\lambda_2}{\alpha\beta\ell n(\lambda_1-\lambda_2)^2},\ \ell>0 \text{ when } \Sigma=\Sigma_i,\ i=1,2, \tag{1.6}$$

problem (1.1)-(1.4) is equivalent to a problem for which the global existence follows from the technique based on Lyapunov functional method (see, e.g., [3], [8], [14], [16], [18], [21] and [24]).

In [12] J. I. Kanel and M. Kirane proved the global existence of solutions for a strongly coupled reaction-diffusion system with homogeneous Neumann boundary conditions and

$$g(u,v) = -f(u,v) = uv^m,\ \ m>0 \ \text{ is an odd integer,}$$

under the conditions

- $0 < a_{22} - a_{11} < a_{21}$,
- $0 < a_{12} \ll 1$,
- $|a_{22} - a_{11} + a_{12} - a_{21}| < \dfrac{\gamma_1+1}{\gamma_1 C_p}$,

where

$$\gamma_1 = \frac{a_{22}-a_{11}-\sqrt{(a_{22}-a_{11})^2+4a_{12}a_{21}}}{2a_{12}} < -1$$

and C_p is the same constant used in Theorem 1 of [20]. Later they improved their results in [13] where they obtained the global existence under the following assumptions

- $a_{22} < a_{11} + a_{21}$,
- $a_{12} < \varepsilon_0 = \dfrac{a_{11}a_{22}(a_{11}+a_{21}-a_{22})}{a_{11}a_{22}+a_{21}(a_{11}+a_{21}-a_{22})}$ if $a_{11} \le a_{22} < a_{11}+a_{21}$,
- $a_{12} < \min\left\{\dfrac{1}{2}(a_{11}+a_{21}), \varepsilon_0\right\}$ if $a_{22} < a_{11}$,

and

- $|F(v)| \le C_F(1+|v|^{1-\varepsilon}), \quad vF(v) \ge 0 \quad \text{for all} \quad v \in \mathbb{R}$,

where $C_F > 0$, ε is any constant such that $0<\varepsilon<1$ and

$$g(u,v) = -f(u,v) = uF(v).$$

On the same direction, S. Kouachi [17] has proved the global existence of solutions for two-component reaction-diffusion systems with a general full matrix of diffusion coefficients, nonhomogeneous boundary conditions and polynomial growth conditions on the nonlinear terms and he obtained in [16] the global existence of solutions for the same system with homogeneous Neumann boundary conditions and

$$g(u,v) = \rho F(u,v), \quad f(u,v) = -\sigma F(u,v), \quad \rho > 0, \quad \sigma > 0,$$

where

- $F(u,v) \leq Ce^{\alpha v^{\beta}}, \; C > 0, \; \alpha > 0, \; 0 < \beta \leq 1, \;$ when $\quad -\mu_2 > \dfrac{\rho}{\sigma},$
- $F(u,v) \leq Ce^{\alpha u^{\beta}}, \; C > 0, \; \alpha > 0, \; 0 < \beta \leq 1, \;$ when $\quad -\mu_2 < \dfrac{\rho}{\sigma},$

under these conditions

- $\|u_0 - \mu_2 v_0\|_\infty < \dfrac{-8\lambda_1\lambda_2\mu_1(\rho + \sigma\mu_2)}{\alpha n\mu_2(\rho + \sigma\mu_1)(\lambda_1 - \lambda_2)^2},$ when $\quad -\mu_2 > \dfrac{\rho}{\sigma},$
- $\|u_0 - \mu_1 v_0\|_\infty < \dfrac{8\lambda_1\lambda_2\mu_2(\rho + \sigma\mu_1)}{\alpha n\mu_1(\rho + \sigma\mu_2)(\lambda_1 - \lambda_2)^2},$ when $\quad -\mu_2 < \dfrac{\rho}{\sigma},$

where μ_1 and μ_2 are the same as in (1.5).

Many chemical and biological operations are described by reaction diffusion systems with a full matrix of diffusion coefficients. The components $u(t,x)$ and $v(t,x)$ can be represent either chemical concentrations or biological population densities (see, e.g., E. L. Cussler [5] and [6]).

We note that the resolution of the problem (1.1)-(1.4) is quite more difficult. As a consequence of the blow-up examples found in [23], we can prove that there is blow-up of the solutions in finite time for such full systems even though the initial data are regular, the solutions are positive and the nonlinear terms are negative, a structure that ensured the global existence in the diagonal case.

Our goal is to understand how the results of the diagonal case extend to the nondiagonal situation without any additional assumption on the diffusion coefficients in the case of possiblity of growth faster than exponential for the reaction terms. For this purpose, we construct the invariant regions in which we can demonstrate that for any initial data in this regions satisfying (1.6), problem (1.1)-(1.4) is equivalent to a problem for which the global existence follows from the same Lyapunov functional used in [24] when the reactive terms satisfies (**A1**)-(**A4**).

Throughout this work, we denote by $\|\cdot\|_p, \quad p \in [1, +\infty)$ the norm in $L^p(\Omega)$ and $\|\cdot\|_\infty$ the norm in $C(\overline{\Omega})$ or $L^\infty(\Omega)$.

2. **Local existence and invariant regions**

The study of local existence and uniqueness of solutions (u, v) of (1.1)-(1.4) follows from the basic existence theory for parabolic semi-linear equations (see, e.g., [2], [9], [11] and [22]). As a consequence, for any initial data in $C(\overline{\Omega})$ or $L^\infty(\Omega)$ there exists a $T^* \in (0, +\infty]$ such that (1.1)-(1.4) has a unique classical solution on $[0, T^*) \times \Omega$. Furthermore, if $T^* < +\infty$, then

$$\lim_{t \uparrow T^*} (\|u(t)\|_\infty + \|v(t)\|_\infty) = +\infty.$$

Therefore, if there exists a positive constant C such that

$$\|u(t)\|_\infty + \|v(t)\|_\infty \leq C \quad \text{for all} \quad t \in [0, T^*),$$

then, $T^* = +\infty$.

Since the initial conditions are in Σ, then under the assumptions (**A1**)-(**A2**), the next proposition says that the classical solution of (1.1)-(1.4) on $[0, T^*) \times \Omega$ remains in Σ for all t in $[0, T^*)$.

Proposition 2.1. *Suppose that the assumptions (**A1**)-(**A2**) are satisfied. Then for any* (u_0, v_0) *in* Σ *the classical solution* (u, v) *of problem* (1.1)-(1.4) *on* $[0, T^*) \times \Omega$ *remains in* Σ *for all* t *in* $[0, T^*)$.

Proof of Proposition 2.1. One starts with the case where $\Sigma = \Sigma_1$. Multiplying equations (1.1) one time through by μ_1 and subtracting (1.2) and another time by $-\mu_2$ and adding (1.2), then if we put

- $z_1 = \mu_1 u - v \quad$ and $\quad z_2 = -\mu_2 u + v \quad$ for all $\quad (u, v) \in \Sigma_1$,
- $F_1 = -\mu_1 f + g \quad$ and $\quad F_2 = -\mu_2 f + g \quad$ for all $\quad (u, v) \in \Sigma_1$,

we get

$$\frac{\partial z_1}{\partial t} - \lambda_1 \Delta z_1 = -F_1(z_1, z_2) \quad \text{in} \quad (0, +\infty) \times \Omega, \tag{2.1}$$

$$\frac{\partial z_2}{\partial t} - \lambda_2 \Delta z_2 = F_2(z_1, z_2) \quad \text{in} \quad (0, +\infty) \times \Omega. \tag{2.2}$$

with the initial conditions:

$$z_i(0, x) = z_i^0(x), \quad i = 1, 2, \quad \text{in} \quad \Omega, \tag{2.3}$$

and the homogeneous boundary conditions:

$$\alpha z_i + (1-\alpha)\frac{\partial z_i}{\partial \nu} = 0, \quad i = 1,2, \quad \text{on} \quad (0,+\infty) \times \partial\Omega, \tag{2.4}$$

Since λ_1 and λ_2 are the eigenvalues of the matrix A^t, (1.1)-(1.4) is equivalent to (2.1)-(2.4). Then to prove that Σ_1 is an invariant region for system (1.1)-(1.2) it is sufficient to prove that the region

$$\left\{(z_1^0, z_2^0) \in \mathbb{R}^2 : \quad z_i^0 \geq 0, \quad i = 1,2\right\} = [0,+\infty)^2 \tag{2.5}$$

is invariant for system (2.1)-(2.2).
Since, from (**A2**), we have $F_1(0,z_2) = 0$ for all $z_2 \geq 0$ and $F_2(z_1,z_2) \geq 0$ for all $(z_1,z_2) \in [0,+\infty)^2$, then we obtain $z_i(t,x) \geq 0,\ i = 1,2$ for all $(t,x) \in [0,T^*) \times \Omega$, thanks to the invariant regions method (see [25]). As a consequence, Σ_1 is an invariant region for (1.1)-(1.2).

For the case $\Sigma = \Sigma_2$, the same reasoning with

- $z_1 = -\mu_1 u + v$ and $z_2 = -\mu_2 u + v$ for all $(u,v) \in \Sigma_2$,
- $F_1 = -\mu_1 f + g$ and $F_2 = \mu_2 f - g$ for all $(u,v) \in \Sigma_2$,

implies the invariance of $[0,+\infty)^2$, and then of Σ_2. □

3. Global existence

Since λ_1 and λ_2 are the eigenvalues of the matrix A^t, then to prove global existence of solutions for problem (1.1)-(1.4) we need to prove it for problem (2.1)-(2.4).

Since we can use the same way to treat the two cases relating to $\Sigma = \Sigma_1$ or $\Sigma = \Sigma_2$, we only deal with the first case.

Since, from (**A2**), we have $F_1 \geq 0$, then z_1 satisfies the maximum principle, *i.e.*,

$$\|z_1(t)\|_\infty \leq \|z_1^0\|_\infty \quad \text{for all} \quad t \in [0,T^*).$$

Based on that, the problem of global existence reduces to establish the uniform boundedness of z_2 in $[0,T^*)$. By L^p-regularity theory for parabolic operator (see, e.g., [19]) it follows that it is sufficient to derive a uniform estimate of $\|F_2(z_1,z_2)\|_p$ on $[0,T^*)$ for some $p > \frac{n}{2}$.

The main result is stated in the following key proposition.

Proposition 3.1. *Suppose that the assumptions (**A1**)-(**A4**) and the restriction* (1.6) *are fulfilled. For every classical solution* (z_1,z_2) *of* (2.1)-(2.4) *on* $[0,T^*) \times \Omega$, *let the function*

$$L : t \longmapsto \int_\Omega \left[\delta z_1 + (M - z_1)^{-\gamma} e^{\alpha p (z_2+1)^\beta}\right](x,t)dx,$$

where α, β, γ, δ, p and M are positive constants such that

$$\beta \geq 1,\ \|z_1^0\|_\infty < M < \frac{2\gamma}{\alpha\beta\ell n} \text{ and } \gamma = \frac{4\lambda_1\lambda_2}{(\lambda_1-\lambda_2)^2}. \tag{3.1}$$

Then there exists $\delta > 0$ and $p > \frac{n}{2}$ such that

$$L \quad \text{is nonincreasing on} \quad [0, T^*). \tag{3.2}$$

Before proving this proposition we first need the following lemma.

Lemma 3.2. *Let (z_1, z_2) be a solution of (2.1)-(2.4) on $[0, T^*) \times \Omega$, then under the assumptions (**A1**)-(**A4**), we have*

$$\int_\Omega F_1(z_1(x,t), z_2(x,t))dx \leq -\frac{d}{dt}\int_\Omega z_1(x,t)dx \tag{3.3}$$

and there exists $\delta_1 > 0$ and $p > \frac{n}{2}$ such that

(3.4)
$$\int_\Omega \left[\alpha p\beta M(z_2+1)^{\beta-1}F_2(z_1,z_2) - \gamma F_1(z_1,z_2)\right] e^{\alpha p(z_2+1)^\beta}dx \leq \delta_1 \int_\Omega F_1(z_1,z_2)dx,$$

where α, β, γ and M are positive constants satisfying (3.1).

Proof of Lemma 3.2. It suffices to integrate the both sides of (2.1) satisfied by z_1 on Ω, to obtain (3.3).
Now, from (3.1), we get $\frac{n}{2} < \frac{\gamma}{\alpha\beta\ell M}$, so we can choose p such that $\frac{n}{2} < p < \frac{\gamma}{\alpha\beta\ell M}$. According to the assumption (**A3**), we have

$$\begin{aligned}&\left[\alpha p\beta M(z_2+1)^{\beta-1}F_2(z_1,z_2) - \gamma F_1(z_1,z_2)\right] e^{\alpha p(z_2+1)^\beta}\\ &\leq \left[\alpha p\beta M(z_2+1)^{\beta-1}\psi(z_2) - \gamma\right] e^{\alpha p(z_2+1)^\beta}F_1(z_1,z_2).\end{aligned}$$

Since $\alpha p\beta\ell M < \gamma$ and $(\eta+1)^{\beta-1}\psi(\eta)$ goes to ℓ as $\eta \to +\infty$, there exists $\eta_0 > 0$ such that for all $\eta > \eta_0$, we obtain

$$\left[\alpha p\beta M(\eta+1)^{\beta-1}\psi(\eta) - \gamma\right] e^{\alpha p(\eta+1)^\beta}F_1(\xi,\eta) \leq 0.$$

On the other hand, if η is in the compact interval $[0, \eta_0]$, then the continuous function

$$\eta \longmapsto \left[\alpha p\beta M(\eta+1)^{\beta-1}\psi(\eta) - \gamma\right] e^{\alpha p(\eta+1)^\beta}$$

is bounded. So that (3.4) immediately follows. □

Proof of Proposition 3.1. Differentiating $L(t)$ with respect to t and using the Green formula, one obtains

$$\frac{d}{dt}L(t) = \delta\frac{d}{dt}\int_\Omega z_1(x,t)dx + I + J, \tag{3.5}$$

where

$$\begin{aligned} I &= \int_{\partial\Omega} \left[\lambda_1\gamma\frac{\partial z_1}{\partial\nu} + \lambda_2\alpha p\beta(M-z_1)(z_2+1)^{\beta-1}\frac{\partial z_2}{\partial\nu}\right](M-z_1)^{-\gamma-1}e^{\alpha p(z_2+1)^\beta}ds \\ &- \int_\Omega \left[\lambda_1\gamma(1+\gamma)|\nabla z_1|^2 + \alpha p\beta\gamma(\lambda_1+\lambda_2)(M-z_1)(z_2+1)^{\beta-1}\nabla z_1\nabla z_2\right. \\ &+ \left.\lambda_2\alpha p\beta(M-z_1)^2\left(\beta-1+\alpha p\beta(z_2+1)^\beta\right)(z_2+1)^{\beta-2}|\nabla z_2|^2\right](M-z_1)^{-\gamma-2}e^{\alpha p(z_2+1)^\beta}dx, \end{aligned}$$

where ds denotes the $(n-1)$-dimensional surface element and

$$J = \int_\Omega \left[\alpha p\beta(M-z_1)(z_2+1)^{\beta-1}F_2(z_1,z_2) - \gamma F_1(z_1,z_2)\right](M-z_1)^{-\gamma-1}e^{\alpha p(z_2+1)^\beta}dx.$$

We now take advantage of (2.4) and $\beta \geq 1$, to obtain that

$$I \leq -\int_\Omega Q(\nabla z_1,\nabla z_2)(M-z_1)^{-\gamma-2}e^{\alpha p(z_2+1)^\beta}dx,$$

where

$$\begin{aligned} Q(\nabla z_1,\nabla z_2) &= \lambda_1\gamma(1+\gamma)|\nabla z_1|^2 + \alpha p\beta\gamma(\lambda_1+\lambda_2)(M-z_1)(z_2+1)^{\beta-1}\nabla z_1\nabla z_2 \\ &\quad +\lambda_2\left(\alpha p\beta(M-z_1)(z_2+1)^{\beta-1}\right)^2|\nabla z_2|^2 \end{aligned}$$

is a quadratic form with respect to ∇z_1 and ∇z_2. The discriminant of Q is given by

$$D = \gamma\left(\alpha p\beta(M-z_1)(z_2+1)^{\beta-1}\right)^2\left[\gamma(\lambda_1-\lambda_2)^2 - 4\lambda_1\lambda_2\right].$$

From conditions (3.1) we have $Q(\nabla z_1,\nabla z_2) \geq 0$ and consequently

$$I \leq 0. \tag{3.6}$$

Concerning the term J, since $0 \leq z_1 \leq \|z_1^0\|_\infty < M$, we observe that

$$J \leq (M-\|z_1^0\|_\infty)^{-\gamma-1}\int_\Omega \left[\alpha p\beta M(z_2+1)^{\beta-1}F_2(z_1,z_2) - \gamma F_1(z_1,z_2)\right]e^{\alpha p(z_2+1)^\beta}dx.$$

Thanks to (3.4), we get $\delta_1 > 0$ such that

$$J \leq \delta_1(M-\|z_1^0\|_\infty)^{-\gamma-1}\int_\Omega F_1(z_1,z_2)dx.$$

Let $\delta = \delta_1(M-\|z_1^0\|_\infty)^{-\gamma-1}$ and using (3.3), we obtain

$$J \leq -\delta\frac{d}{dt}\int_\Omega z_1(x,t)dx. \tag{3.7}$$

From (3.5)-(3.7), we conclude that

$$\frac{d}{dt}L(t) \leq 0.$$

This concludes the proof of Proposition 3.1. □

We can now establish the main result of this article.

Theorem 3.3. *Under the assumptions (**A1**)-(**A4**), the classical solutions of* (1.1)-(1.4) *with initial data in* Σ_1 *satisfying* (1.6) *are global and uniformly bounded on* $[0, +\infty) \times \Omega$.

Proof of Theorem 3.3. Let p be the same as in Proposition 3.1. Since $M^{-\gamma} \leq (M - \xi)^{-\gamma}$ for all $\xi \in [0, \|z_1^0\|_\infty]$, it follows that

$$\|F_2(z_1, z_2)\|_p^p = \int_\Omega |F_2(z_1, z_2)|^p dx \leq M^\gamma K^p L(t)$$

where

$$K = \max_{0 \leq \xi \leq \|z_1^0\|_\infty} \varphi(\xi).$$

By Proposition 3.1, we deduce

$$\|F_2(z_1, z_2)\|_p^p \leq M^\gamma K^p L(0).$$

Consequently, $F_2(z_1(t, .), z_2(t, .))$ is uniformly bounded in $L^p(\Omega)$ for all $t \in [0, T^*)$ with $p > \frac{n}{2}$. Using the regularity results for solutions of parabolic equations in [19], we conclude that the solutions of the problem (1.1)-(1.4) are uniformly bounded on $[0, +\infty) \times \Omega$. $\square$

Remark 3.4. *When ℓ is a nonnegative constant, we can replace the restriction* (1.6) *by*

$$\ell \|\mu_i u_0 - v_0\|_\infty < \frac{8\lambda_1\lambda_2}{\alpha\beta n(\lambda_1 - \lambda_2)^2} \quad \textit{when} \quad \Sigma = \Sigma_i,\ i = 1, 2,$$

and we observe that if $\ell = 0$, then the initial conditions in Σ are given arbitrarily.

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